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Characterizing CDMA downlink feasibility via effective interference

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Characterizing CDMA Downlink Feasibility via Effective Interference *†

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Abstract

This paper models and analyses downlink power assignment feasibility in Code Division Multiple Access (CDMA) mobile networks. By discretizing the area into small segments, the power requirements are characterized via a matrix representation that separates user and system characteristics. We obtain a closed-form analytical expression of the so-called Perron-Frobenius eigenvalue of that matrix, which provides a quick assessment of the feasibility of the power assignment for each distribution of calls over the segments. Although the obtained relation is non-linear, it basically provides an effective interference characterisation of downlink feasibility. Our results allow for a fast evaluation of outage and blocking probabilities, and enable a quick evaluation of feasibility that may be used for Call Acceptance Control.

Keyword: CDMA, downlink, discretized cells, Perron-Frobenius eigenvalue, feasible power, outage, blocking.

AMS Subject Classification: Primary: 90B18, 90B22; Secondary: 60K25

1 Introduction

The third generation Universal Mobile Telecommunications System (UMTS) employes Code Division Multiple Access (CDMA) as the technique of sharing the network capacity among users. In a CDMA system, calls share a common spectrum, their transmissions are separated using (pseudo) orthogonal codes. The impact of multiple calls is an increase in the interference level, that limits the capacity of the system. The assignment of transmission powers to calls is an important problem for network operation, since the interference caused by a call is directly related to that power.

In a CDMA system the uplink (mobile terminal to Base Transmitter Station (BTS)) and downlink (BTS to mobile) have different characteristics, and must be analysed separately. The uplink determines coverage, whereas the downlink determines capacity. As the downlink has more capacity (due to e.g. a higher transmit power of the BTSs), in many studies the uplink has been investigated in detail. A complicating factor is that the actual location within a cell is important for characterising CDMA system capacity (see e.g. [LB03]). Most studies therefore consider homogeneous traffic loads. A typical capacity measure is the pole capacity [Sip02]. A successful analytical uplink concept is the effective interference model developed by [EE99], which enables a fast evaluation of network state feasibility. However, the analysis in [EE99]

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indeed requires a homogeneous distribution of the users over the network cells. In [Han99], feasibility is characterised via the Perron-Frobenius eigenvalue of an interference matrix of the network state. Unfortunately, for the uplink the PF eigenvalue is not available in closed-form so that it provides only a semi analytical evaluation of the uplink capacity. For the downlink most studies are based on discrete event or Monte-Carlo simulation, see e.g. [Sta02], leading to slow evaluation of feasibility and/or capacity. The objective of this paper is to develop an analytical model that allows a fast evaluation of the downlink feasibility of CDMA under non-homogeneous traffic load. We aim for an analog of the uplink effective interference model.

We focus on modeling BTSs located along a highway to include both non-homogeneity of the call distribution, and mobility of calls. Users are located in cars passing through the cells. Due to e.g. traffic jams ("hot spots") the load of the cells will not be distributed evenly along the road. To characterize the distribution of a single type of calls in the cells, we propose a discretized-cell model. Each cell is divided into small segments. Then, the nonhomogeneous load can be characterized by the mean number of calls and fresh call arrival rates in the segments. Taking into account interference between segments in neighboring cells and between segments within the cells, we express the generated downlink interference per segment towards the other segments in a matrix form. The resulting matrix characterizes the feasibility of each call configuration, which can be determined by investigating the Perron-Frobenius (PF) eigenvalue of the matrix. Furthermore, a state space of feasible call configurations over the segments is defined, and two performance measures, the outage and blocking probability, are derived from our model. The model is also used to determine the optimal cell border in downlink CDMA. Our results are illustrated by some numerical examples.

The remainder of this paper is organized as follows. Section 2 describes the downlink interference model. The performance analysis is presented in Section 3. In Section 4 we present the numerical results, and, finally, in Section 5 we summarize our work and draw conclusions.

2 Model

This section describes our downlink interference model. First, for persistent users we describe a discretized effective interference model. Then, for non-persistent users a stochastic model taking into account a Poisson arrival process of fresh calls, a random call length distribution, and mobility is presented. For simplicity, as we are primarily interested in the interaction between mobility of users along a road, and the teletraffic behaviour of our wireless network, we focus on a two cell model, where only the area in between two base stations is taken into account. The description can readily be generalised to larger networks.

2.1 Interference model: persistent calls

Consider a linear network of two base transmitter stations (BTSs) X and Y, say. Let the area between these BTSs be divided into segments of length δ. For the description below, we fix the radii of the cells. Let cell X resp. Y contain I resp. J segments, labelled $i = 1, ..., I$, $j = 1, ..., J$, respectively. Then $L_1 = I\delta$ is the radius of cell X, see Figure 1. Let $D = I + J$ the distance between the BTSs. We assume that the segments are small, so that we may locate all subscribers in a segment in the middle of that segment, i.e. for segment i of cell X, users are located at distance $i^* = \delta\left[(i-1)+i\right]/2$ from X.

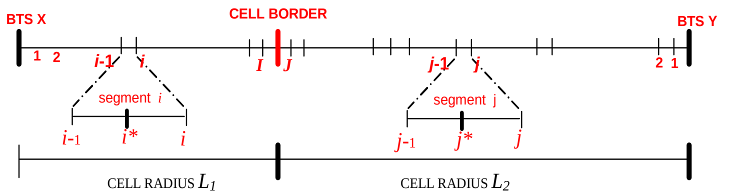

Fig. 1. Discretised BTSs Model

Let X_i resp. Y_j be the transmit power of BTS X resp. Y to user x_i resp. y_j in segment i resp. j (located at point i^* resp. j^*) of cell X resp. Y. A common measure for the quality of the transmission is the *energy per bit to interference ratio*, $\left(\frac{E_b}{I_0}\right)_i$, that for user x_i is defined as (see e.g. [EE99])

$$\begin{aligned}\left(\frac{E_b}{I_0}\right)_{x_i} &= \frac{W}{R}\frac{\text{useful signal power received by user } x_i}{\text{interference power + thermal noise}} \\ &= \frac{W}{R}\frac{X_i\,(i^*)^{-\gamma}}{(D-i^*)^{-\gamma}\sum_{j=1}^{J} m_j Y_j \;+\alpha\,(i^*)^{-\gamma}\left(\sum_{l=1}^{I} n_l X_l - X_i\right) + N_0}\end{aligned} \tag{1}$$

where W is the system chiprate, R is the data rate, n_i resp. m_j is the number of users in segment i resp. j of cell X resp. Y, α is non-orthogonality factor, γ is the path loss exponent, and N_0 is the thermal noise level. We assume that all users have the same data rate R.

Transmission at sufficient quality requires the energy per bit to interference ratio to exceed a threshold ϵ^*, i.e., user x_i requires BTS X to transmit enough power so that $\left(\frac{E_b}{I_0}\right)_{x_i} \geqslant \epsilon^*$. Rearranging terms in (1), user x_i requires that

$$X_i \geqslant \Gamma\left(\frac{D-i^*}{i^*}\right)^{-\gamma}\sum_{j=1}^{J} m_j Y_j + \alpha\Gamma\left(\sum_{l=1}^{I} n_l X_l - X_i\right) + \Gamma N_0\left(\frac{1}{i^*}\right)^{-\gamma} \tag{2}$$

where $\Gamma = \epsilon^*\frac{R}{W}$. By analogy, user y_j located in segment j of cell Y requires a transmitted power Y_j such that

$$Y_j \geqslant \Gamma\left(\frac{D-j^*}{j^*}\right)^{-\gamma}\sum_{i=1}^{I} n_i X_i + \alpha\Gamma\left(\sum_{l=1}^{J} m_l Y_l - Y_j\right) + \Gamma N_0\left(\frac{1}{j^*}\right)^{-\gamma} \tag{3}$$

To enhance readability, the system of equations (2), (3), $i = 1, 2, \cdots, I$, $j = 1, 2, \cdots, J$, can readily be stated in matrix form. To this end, let $\mathbf{X} = \begin{pmatrix} X_1 & X_2 & \cdots & X_I \end{pmatrix}^T$, $\mathbf{Y} = \begin{pmatrix} Y_1 & Y_2 & \cdots & Y_J \end{pmatrix}^T$ represent the transmit powers to the segments; $\mathbf{N}=diag(n_i)$ the diagonal matrix of size $I \times I$ that represents the number of users in each segment in cell X, and similarly $\mathbf{M}=diag(m_i)$ is a diagonal matrix of size $J \times J$ that represents the number of users in each segment in cell Y; $\mathbf{P}_X=(p^X_{ij})$, where $p^X_{ij} = \left(\frac{D-i^*}{i^*}\right)^{-\gamma}$ the matrix of size $I \times J$ that represents the inter-cell path loss, i.e., the path loss from segment j in cell Y to segment i in cell X, and similarly $\mathbf{P}_Y=(p^Y_{ji})$, where $p^Y_{ji} = \left(\frac{D-j^*}{j^*}\right)^{-\gamma}$; $\mathbf{D}_X = \begin{pmatrix} \left(\frac{1}{1^*}\right)^{-\gamma} & \left(\frac{1}{2^*}\right)^{-\gamma} & \cdots & \left(\frac{1}{I^*}\right)^{-\gamma} \end{pmatrix}^T$, $\mathbf{D}_Y = \begin{pmatrix} \left(\frac{1}{1^*}\right)^{-\gamma} & \left(\frac{1}{2^*}\right)^{-\gamma} & \cdots & \left(\frac{1}{J^*}\right)^{-\gamma} \end{pmatrix}^T$ representing the intra-cell path loss; $\mathbf{I}_X$ be the identity matrix of size $I \times I$, and $\mathbf{I}_Y$ the identity matrix of size $J \times J$.

Then the system of equations (2), (3) reads

$$\begin{cases} (1+\alpha\Gamma)\mathbf{I}_X\mathbf{X} \geqslant & \Gamma\mathbf{P}_X\mathbf{M}\mathbf{Y} + \alpha\Gamma\mathbf{N}\mathbf{X} + \Gamma N_0\mathbf{D}_X \\ (1+\alpha\Gamma)\mathbf{I}_Y\mathbf{Y} \geqslant & \Gamma\mathbf{P}_Y\mathbf{N}\mathbf{X} + \alpha\Gamma\mathbf{M}\mathbf{Y} + \Gamma N_0\mathbf{D}_Y \end{cases} \tag{4}$$

Furthermore, this system can be written as

$$(s\mathbf{I} - \mathbf{T})\mathbf{Z} \geqslant \mathbf{c} \tag{5}$$

where
$s = (1+\alpha\Gamma)$; $\mathbf{I} = \begin{pmatrix} \mathbf{I}_X & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_Y \end{pmatrix}$; $\mathbf{T} = \begin{pmatrix} \alpha\Gamma\mathbf{1}_X\mathbf{N} & \Gamma\mathbf{P}_X\mathbf{M} \\ \Gamma\mathbf{P}_Y\mathbf{N} & \alpha\Gamma\mathbf{1}_Y\mathbf{M} \end{pmatrix}$; $\mathbf{Z} = \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix}$;
$\mathbf{c} = N_0\Gamma \begin{pmatrix} \mathbf{D}_X \\ \mathbf{D}_Y \end{pmatrix}$ and $\mathbf{1}_X$ ($\mathbf{1}_Y$) is a matrix of size $I \times I$ ($J \times J$) with all entries equal to 1.

2.2 Interference model: non-persistent and moving calls

Consider the linear wireless network covering a road as depicted in Figure 2, but now with non-persistent and moving users, where fresh calls arrive according to a Poisson arrival process with rate proportional to the density of users along the road, and where users move along the road according to the laws of road traffic movement.

The prediction of the location of subscribers used in this paper requires an estimate of the density of subscribers. For the purpose of this paper, a simplified model as provided in [New93] is sufficient. Let $k(x,t)$ denote the density of subscribers at location x at time t. Then the traffic mass conservation principle states that $\frac{\partial k(x,t)}{\partial t} + \frac{\partial k(x,t)v(x,t)}{\partial x} = 0$, where $v(x,t)$ is the velocity on location x at time t.

In a mobile network the number of subscribers making a call is typically substantially smaller than the number of subscribers not making a call. Therefore, it is natural to assume that *fresh calls* in segment s are generated according to a Poisson process with non-stationary arrival rate

$$\lambda_s(t) := \alpha \int_{r_s}^{r_{s+1}} k(x,t)dx \tag{6}$$

proportional to the density of traffic in segment s at time t, where α is the arrival rate of fresh calls per unit traffic mass, and r_s and r_{s+1} are the borders of segment s. Let the call lengths be independent and identically distributed random variables, with common distribution G and mean τ independent of the location and traffic density.

3 Performance analysis

In this section, we first establish feasibility for persistent calls via the Perron-Frobenius eigenvalues of the matrix $\mathbf{T}$, that is explicitly provided in Section 3.1. Section 3.2 considers the model with non-persistent flows and discusses the time-dependent distribution of calls over the segments, and corresponding blocking and outage probabilities.

3.1 Persistent calls: feasibility

Feasibility of our cellular system is characterised by (5), where the distribution of calls over the segments appears in $\mathbf{T}$. Notice that the system and user characteristics in this matrix can be separated:

$$\mathbf{T} = \begin{pmatrix} \alpha\Gamma\mathbf{1}_X & \Gamma\mathbf{P}_X \\ \Gamma\mathbf{P}_Y & \alpha\Gamma\mathbf{1}_Y \end{pmatrix} \begin{pmatrix} \mathbf{N} & 0 \\ 0 & \mathbf{M} \end{pmatrix} = \mathbf{SU} \tag{7}$$

where $\mathbf{U} = \begin{pmatrix} \mathbf{N} & 0 \\ 0 & \mathbf{M} \end{pmatrix}$, represents the distribution of the number of calls in each segment, and $\mathbf{S}$=$\begin{pmatrix} \alpha\Gamma\mathbf{1}_X & \Gamma\mathbf{P}_X \\ \Gamma\mathbf{P}_Y & \alpha\Gamma\mathbf{1}_Y \end{pmatrix}$ contains the system parameters. Notice that the entries of $\mathbf{S}$ are fixed for given system parameters.

Under the assumption of perfect power control, if $(s\mathbf{I} - \mathbf{T})\mathbf{Z} \geqslant \mathbf{c}$ then the equation is satisfied with equality, i.e., $(s\mathbf{I} - \mathbf{T})\mathbf{Z} = \mathbf{c}$. According to the Perron-Frobenius theorem in [Sen73], feasibility is then determined by the Perron-Frobenius (PF) eigenvalue λ of matrix $\mathbf{T}$,

$$\mathbf{Z} = (s\mathbf{I} - \mathbf{T})^{-1}\mathbf{c} \Longleftrightarrow s > \lambda. \tag{8}$$

Clearly, as the system parameters are fixed, λ is completely determined by the distribution of calls over the segments, i.e., $\lambda = \lambda(\mathbf{T}) = \lambda(\mathbf{U})$.

The characterisation (8) provides a clear motivation for the discretisation into segments as we obtain a downlink interference model that is very similar to uplink models such as studied in [BCP00], [EE99], [Han99], [Zan92], where feasibility of the uplink power control algorithm is characterised via the Perron-Frobenius eigenvalue of a matrix containing the number of calls in the cell (not in segments). Effective interference models such as developed in [EE99] allow for a characterisation of feasibility based on that total number only, but they assume a homogeneous distribution of calls over the area covered by a cell.

Tedious and lengthy calculations of the characteristic polynomial of $\mathbf{T}$ provide an explicit expression of λ in terms of the number of calls in the segments. These calculations are omitted.

Theorem 1: The Perron-Frobenius eigen value of $\mathbf{T}$ is

$$\lambda(\mathbf{N},\mathbf{M}) = \frac{\Gamma}{2}\alpha\left(\sum_{i=1}^{I} n_i + \sum_{j=1}^{J} m_j\right) + \frac{\Gamma}{2}\sqrt{\alpha^2\left(\sum_{i=1}^{I} n_i - \sum_{j=1}^{J} m_j\right)^2 + 4\sum_{i=1}^{I} p_i n_i \sum_{j=1}^{J} p_j m_j} \tag{9}$$

Feasibility of a user configuration $\mathbf{U}$ is now readily determined by checking the inequality $\lambda(\mathbf{U}) < s$. The set of all feasible user configurations is

$$\mathcal{S} = \left\{\mathbf{U} \mid \lambda(\mathbf{U}) < s, \quad \mathbf{U} = (\mathbf{N},\mathbf{M}) \in \mathbb{N}^{I+J}\right\} \tag{10}$$

It can readily be shown that $\mathcal{S}$ is a coordinate convex set, so that we may invoke the theory of loss networks [Ros95] to characterise the distribution of non-persistent calls, which is the topic of the next section.

3.2 Non-persistent calls: outage and blocking probabilities

We may distinguish two ways of handling fresh calls that bring the system in a non-feasible state: we may either block and clear the call from the system (*fresh call blocking*), or accept the call in which case the system is said to be in outage (*outage probability*) and (some) calls do not reach their energy per bit to interference threshold ϵ^*, until completion of some (other) call. These 'outage' and 'blocking' cases lead to different stochastic processes recording the number of calls in the segments.

When calls are blocked and cleared when the state is not feasible, the set of feasible states is the finite set $\mathcal{S}$ as defined in (10). Let $\{X(t), t \geq 0\}$ be the stochastic process recording the number of non-persistent and moving calls over the segments, which takes values in the finite state space $\mathcal{S}$. A state of the stochastic process is a vector $\mathbf{U} = (n_1, n_2, \cdots, n_I, m_J, \cdots, m_2, m_1)$, that will be labelled as $\mathbf{U} = (u_1, u_2, \cdots, u_I, u_{I+1}, \cdots, u_{I+J})$. When calls are not blocked, but instead all (or some) calls are in outage when the system state is not feasible, then all vectors in the positive orthant

$$\mathcal{S}^\infty = \left\{ \mathbf{U} \mid \mathbf{U} = (\mathbf{N}, \mathbf{M}) \in \mathbb{N}^{I+J} \right\} \tag{11}$$

are possible system states. Let $\{X^\infty(t), t \geq 0\}$ be the corresponding stochastic process.

We are primarily interested in the distribution of calls over the segments $P\left(X^\infty(t) = \mathbf{U}\right)$, and $P\left(X(t) = \mathbf{U}\right)$. For the 'outage case' this distribution can be evaluated in closed form:

$$P\left(X^\infty(t) = \mathbf{U}\right) = \prod_{s=1}^{I+J} e^{-\rho_s^\infty(t)} \frac{\rho_s^\infty(t)^{u_s}}{u_s!}, \tag{12}$$

where

$$\rho_s^\infty(t) = \tau \lambda_s(t) \tag{13}$$

is the time-dependent *load offered* to segment s: the distribution of the number of calls in cell s is Poisson with mean $\rho_s^\infty(t)$ proportional to the density of traffic and insensitive to the distribution of the call length G except through its mean τ, see [MW93] for a general framework for networks with unlimited capacity, and [UB01] for a derivation of the insensitivity result (12).

For the 'blocking case' the distribution $P\left(X(t) = \mathbf{U}\right)$ cannot be obtained in closed form. However, for the regime of small blocking probabilities, $P\left(X(t) = \mathbf{U}\right)$ can be adequately approximated using the Modified Offered Load (MOL) approximation:

$$P\left(X(t) = \mathbf{U}\right) \approx P\left(X^\infty(t) = \mathbf{U} \mid X^\infty(t) \in \mathcal{S}\right) = \prod_{s=1}^{I+J} e^{-\rho_s^\infty(t)} \frac{\rho_s^\infty(t)^{u_s}}{u_s!} \Bigg/ \sum_{u \in \mathcal{S}} \prod_{s=1}^{I+J} e^{-\rho_s^\infty(t)} \frac{\rho_s^\infty(t)^{u_s}}{u_s!}$$

The approximation is exact for a loss network in equilibrium. For networks with time-varying rates the MOL approximation is investigated in [MW94] for the Erlang loss queue, and is applied to networks of Erlang loss queues in [AB00]. It is shown that the error of the MOL approximation is decreasing with decreasing blocking probabilities and with decreasing variability of the arrival rate.

Outage and blocking probabilities are now readily obtained. First consider the 'outage case'. As the number of calls in the system increases, all calls suffer a gradual degradation of their QoS. If the energy per bit to interference ratio of a call falls below its target value ϵ^*, then the system is said to be in outage. The outage probability, $P_{out} = P\left(X^\infty(t) \notin \mathcal{S}\right)$, is defined as the probability that an (instant) outage occurs to the system. The outage probability of a user in segment j in a cell can be formulated as follows :

$$P_{out} = P\left(\epsilon_j < \epsilon^* \text{ for some j}\right) = \Pr\left(\lambda\left(X(t)\right) > s\right) \tag{14}$$

The outage probability cannot be evaluated in closed form due to the complexity of the feasible set $\mathcal{S}$, and will be evaluated via Monte-Carlo simulation.

For the 'blocking case', the fresh call blocking probability must be determined per segment. To this end, define the blocking set of segment k as $\mathcal{S}_k = \{\mathbf{U} \in \mathcal{S} \mid \lambda\left(\mathbf{U} + \mathbf{e}_k\right) > s\}$ where $\mathbf{e}_k$ is the unit vector with entry k equal 1, and all other entries 0. Then, as is shown in [AB01], the blocking probability, $B_k(t)$, of a segment k at time t is approximated as

$$B_k(t) \approx P\left(X^{\infty}(t) \in \mathcal{S}_k \mid X^{\infty}(t) \in \mathcal{S}\right) = \sum_{\mathbf{U}\in\mathcal{S}_k} \prod_{s=1}^{I+J} e^{-\rho_s^{\infty}(t)} \frac{\rho_s^{\infty}(t)^{u_s}}{u_s!} \Bigg/ \sum_{u\in\mathcal{S}} \prod_{s=1}^{I+J} e^{-\rho_s^{\infty}(t)} \frac{\rho_s^{\infty}(t)^{u_s}}{u_s!}$$

The blocking probability cannot be evaluated in closed form due to the complexity of the feasible set $\mathcal{S}$, and will be valuated via Monte-Carlo simulation.

4 Numerical results

In this section, we use our analytical expressions to investigate blocking and outage for a system with a fixed border. Then we investigate the optimal location of the downlink border between two cells. The distance between the two BTSs X and Y is 2000 meter, which is divided into 40 segments of width 50 meter. Throughout this section, we assume that a block shape traffic jam of width 10 segments moves from BTS X to BTS Y at constant speed, see Figure 2. The background load in segments not covered by the hot spot is $\rho_s = 1$ Erlang, whereas the load in segments inside the hot spot is 12 Erlangs.

We will investigate both results from Monte-Carlo simulation, and a prediction based on the Perron-Frobenius eigenvalue for the load in the segments. Sufficient samples are generated to have 95% confidence and 10% relative precision. Throughout this section, the parameters of our Wideband-CDMA system are those provided in [HT00]: the system chiprate $W = 3.84$ MHz, the required (for voice) $\epsilon^* = 5$ dB, the non orthogonality factor $\alpha = 0.3$, and the path loss exponent $\gamma = 4$. These values result in $s = 1.01$.

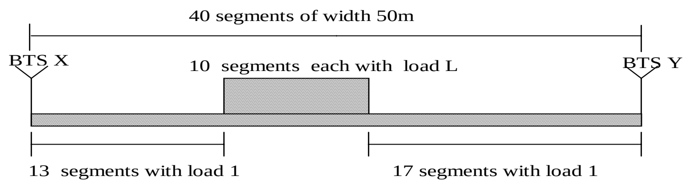

Fig.2. Traffic Load with rectangular shape of hot spot

To facilitate a graphical representation of our results, we will depict blocking probabilities only for those time instances at which the hot spot enters a new segment. Typical locations of the hot spot that will be depicted are the situation of Figure 2, where left-hand side of the hot spot is located 13 segments from BTS X. The location of the hot spot at $12 + i$ segments from BTS X will be referred to as type i traffic load, i.e. Figure 2 depicts type 1 traffic.

4.1 Fixed Border

First consider the commonly studied case of a fixed border located in the middle between the BTSs, i.e. each cell consists of 20 segments. A fast evaluation of feasibility is obtained by merely inserting the load vector into (9), i.e., assuming a deterministic number of calls $\rho_s = 12$ in the segments of the hot spot. Figure 3 depicts the PF eigenvalue for this case (deterministic), and also depicts that value obtained from a Monte-Carlo simulation of the number of calls in the segments for the 'blocking case' (mean and upper and lower bounds of the confidence interval are depicted). We see that λ reaches its maximum when the hot spot is located in the middle between the BTSs (type 3). Furthermore, the approximative value for λ obtained by using the load vector lies well inside the confidence interval, and, as expected, slightly over-estimates the value obtained from our simulation study. As the criterion for feasibility is $\lambda < s$, this yields a conservative approximation of feasibility.

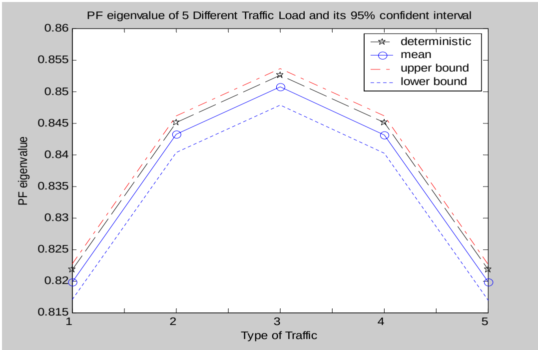

Fig. 3. The Perron-Frobenius eigenvalue

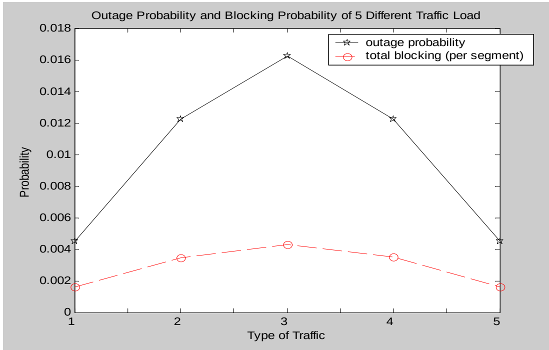

Fig.4. Outage and total blocking

Blocking and outage probabilities can be obtained via Monte-Carlo simulation. For the cases studied above, Figure 4 depicts these probabilities. Clearly, the shape of these curves follows that of λ, which is intuitively obvious. Also, the outage probability exceeds the total blocking (probability that a fresh call is blocked irrespective of its location). The outage probability does not discriminate between segments. Below we will numerically investigate the blocking probabilities per segment for the moving hot spot.

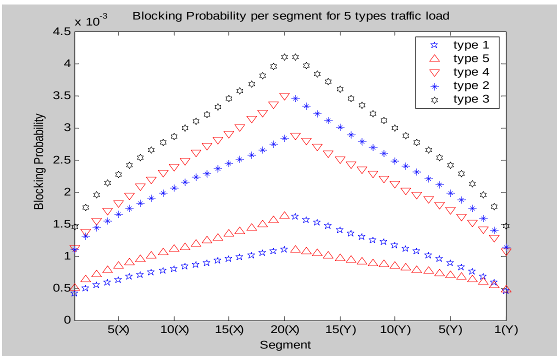

Fig. 5. Blocking probability per segment

Figure 5 depicts blocking probabilities per segment for traffic types 1 – 5. As the traffic load is located more to the left, the blocking probability of the segment in the right is higher (see type 1 and type 2 traffic load). Furthermore, as the traffic load in the right cell is higher, the blocking probability of the segment in the left is higher (see type 4 and type 5 traffic load). While in type 3, the blocking probabilities of both cells are equal since the traffic load is symmetric.

4.2 Optimal Border Location

Let us now investigate the optimal location of the border between the cells. We have increased the load per segment in the hot spot to 17 Erlangs, and we will fix the traffic to be type 1. Figure 6 depicts the deterministic approximation of λ based on the offered load only. The graph has a clear peak for a cell border between roughly 750 and 1150 meters from BTS X. From the curve it seems optimal for the cell border to be such that the entire hot spot resides in a single cell. Monte-Carlo simulation of the blocking probabilities per segment for type 1 traffic and different locations of the border at 700, 900, 1000 and meters from BTS X as depicted in Figure 7 also support that claim. For the case of a single block shape hot spot we have analytically investigated the optimal location of the cell border. Indeed, first results indicate that for this case the Perron-Frobenius eigenvalue has a (local) minimum when all calls reside in a single cell. Whether this is true in general remains a topic for further research.

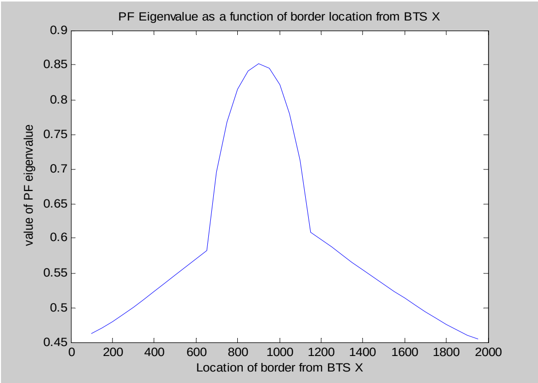

Fig. 6. Perron-Frobenius eigenvalue versus border location

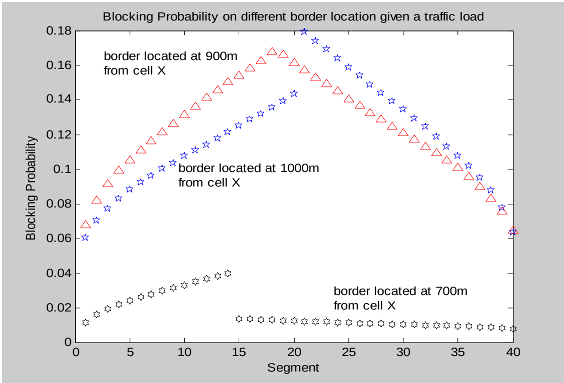

Fig. 7. Blocking per segment versus border location

5 Conclusion

This paper has provided a first step towards a downlink effective interference model. We have obtained an explicit decomposition of system and user characteristics, and have provided an explicit analytical expression for the Perron-Frobenius eigenvalue that determines feasibility and blocking probabilities. Second, based on this result we have numerically investigated blocking probabilities and found for the downlink the surprising result that indicates that it is best to allocate all calls to a single cell. Clearly, this is not feasible under power constraints, and also the uplink that determines coverage should be taken into account. It is among our aims for further research to investigate the optimal cell border based on both uplink and downlink interference invoking e.g. the results of [Sab02] to characterise the optimal uplink border.

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